Review for Test 2

First of all you should spend a little time going over the last test. Mathematics is cumulative so you have to keep going back and reviewing. For example it is certainly possible that there might be a question that would combine using both the microscope equation with a rate equation. You should also read §3.1–§3.7 (pages 89–156) and especially §3.2, §3.3, §3.4 and §3.5. As reading mathematics is hard work it is often best to do the reading in small groups and take turns explaining things to each other. (I remark this is also the way that many practicing scientists work. (When I am working on understanding a new paper, I often work at a black board with some else interested in the same thing and we work through the details together.) Not only is this less boring, you will remember what you have done much longer.)

Since the last test the main new concept is that of the derivative. If $f(x)$ is a function the derivative at $x = a$ is defined by

$$f'(a) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}.$$

(The book changes Δx to h and writes this as $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.) In practice derivatives are rarely computed directly form this definition as there are easy to use formulas for them. The formulas that we have to date are

function	derivative
c	0
cx	c
cx^p	pcx^{p-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
b^x	$(\ln b)b^x$
$cf(x)$	$cf'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

You should enough drill on taking derivatives that it comes easily. This is important as at lest 25% of the test will be taking derivatives. And some of the problems that do not directly ask for derivatives will still involve finding derivatives in their solution. You should make sure that you can do all the derivative problems I given you on the hand outs. We have also computed, or at least approximated, the derivative $f'(a)$ by choosing two points (x_1, y_1) and (x_2, y_2) on the graph of $y = f(x)$ close to the point $(a, f(a)$, computing $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ and then using $f'(a) \approx \Delta y / \Delta x$. For example if the function is given as a graph or as a table of values, then this is how one has to approximate $f'(a)$. For problems related to the derivative go over the old quizzes, and look at the following problems form the text #1–#3, #8, #9 on pages 102–105, #4, #10, #12, #13 #18, #19 pages 117–120 (note that when the book says to use a computer microscope you can just use a difference

$$\frac{f(a+h) - f(a)}{h} \quad \text{or better yet} \quad \frac{f(a+h) - f(a-h)}{2h}$$

to approximate $f'(a)$ by taking h small (say $h = .001$). Thus you do not need a computer, only a calculator to do these problems.)

Closely related to the derivative is the is the ***microscope equation***. At the point $x = a$ the microscope equation is

$$\Delta y \approx f'(a)\Delta x.$$

If we change notation just a little and write $\Delta x = x - a$ and $\Delta y = y - f(a)$ then the microscope equation becomes

$$y - f(a) \approx f'(a)(y - f(a)).$$

It is important to note all that is involved in finding the microscope equation is computing $f'(x)$ and then setting $x = a$. For example
Sample problem: *If $V = s^3 + 2s$ then find the microscope equation at $s = 2$.*
Solution: First find $V'(s) = 3s^2 + 2$. Then $V'(2) = 3 \cdot 2^2 + 2 = 14$. Thus the microscope equation is

$$\Delta V \approx 14\Delta s.$$

done!

We have used the microscope equation in a large number of problems. The basic idea is that given any two of the pieces Δy, Δx or $f'(a)$ we can solve for (or approximate) the third one. It is also used in error analysis. As examples related to this look at the following problems form the text pages 125–127 #2, #3, #4, #5, #8.

Here are some other problems that you should look at. Pages 134–136 #3, #4, #5(Your **will** have to do this type of graphing) #6, #15. Pages 145–147 #1, #2, #13

The last topic we have covered is partial derivatives and the full microscope equation in §3.7. You will be expected to be able to find partial derivatives and do easy problems with the full microscope equation. As practice look at pages 152–154 #1, #3, #7, #8, #9.