Mathematics 122 Final exam Name: ______________________________

You are to use your own calculator, no sharing.

Show your work to get credit. There is room for scratch work on the last page.

(1) (20 points) Compute the following antiderivatives:

(a) $\int (6x^2 + 8x - 5)\, dx =$ ____________________

(b) $\int \left(6\sqrt{t} - \frac{12}{t^2}\right)\, dt =$ ____________________

(c) $\int \frac{6}{u}\, du =$ ____________________

(d) $\int 3e^{5r}\, dr =$ ____________________

(2) (10 points) Compute the following definite integrals:

(a) $\int_0^a x(a-x)\, dx$ ____________________

(b) $\int_1^b 6e^{2q}\, dq$ ____________________

(3) (5 points) A tank holding 500 gallons of water springs a leak such that the rate the water is leaking t hours after the leak starts is $5 + (.03)t^2$ gallons per hour. Give a formula for the amout of water in the tank n hours after the leak starts.

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(4) (5 points) Let $F(t)$ be a function such that $F'(t) = 1 + 6t^2$ and $F(-1) = 3$. Then find $F(t)$

$F(t) =$ ____________________

(5) (5 points) Compute the following using your calculators.

(a) $\displaystyle\int_{-1}^{2} \frac{x+1}{2^{x+3}}\,dx =$ ________________

(b) $f'(4.7)$ where $f(t) = (2+t)3^{t+1}$ $\qquad f'(4.7) =$ ________________

(6) (30 points) Compute the following derivatives. Assume that a, b, and c are constants.

(a) $y = 5x^4 - 4x^3 + 3x^2 + 2x + 1$ $\qquad y' =$ ________________

(b) $f(x) = 3\sqrt{x} - \dfrac{9}{x}$ $\qquad f'(x) =$ ________________

(c) $C = 4e^q + 5\ln q$ $\qquad \dfrac{dC}{dq} =$ ________________

(d) $w = ae^{4z}$ $\qquad \dfrac{dw}{dz} =$ ________________

(e) $y = -7(3x+1)^9$ $\qquad y' =$ ________________

(f) $f(x) = 8e^{5x^3+x}$ $\qquad f'(x) =$ ________________

(g) $A(r) = 4\sqrt{r^3+1}$ $\qquad A'(r) =$ ________________

(h) $y = 3x^2e^{x^3}$ $\qquad y =$ ________________

(i) $P = 100(1.05)^t$ $\qquad \dfrac{dP}{dt} =$ ________________

(j) $q = \dfrac{e^t}{2+e^t}$ $\qquad \dfrac{dq}{dt} =$ ________________

(7) (5 points) Find the inflextion points, if any, of $y = 2x^3 + 24x^2 + x + 1$. Give both the x and y coordinates.

____

(8) (5 points) Find the tangent line to $y = 5 - x^2$ at the point where $x = -1$.

____

(9) (5 points) Draw graphs of functions with the following properties

(a) f is increasing at an increasing rate.

(b) $f' > 0$ and $f'' < 0$.

(c) $f(1) = 2$, $f'(1) = -1$ and f has an inflection point where $x = 1$.

(10) (5 Points) For a function given by the table

q	3.0	3.5	4.0	4.5
$C(q)$	12.1	12.8	13.9	15.4

(a) Make a table for $C'(t)$.

(b) Make a table for $C''(t)$.

(11) (5 points) A turkey put in the oven to cook. Let $T(t)$ be temperature of the turkey in degrees Fahrenheit t minutes after it was put in the oven. Assume that $T(25) = 125$ and that $T'(25) = 2.5$

(a) Explain why $T'(20)$ is positive.

(b) What are the units on $T(25)$? ____________________

(c) What are the units on $T'(25)$? ____________________

(d) Estimate $T(27)$. $T(27) \approx$ ____________________

(12) (5 points) A group of students decide to sell study guides for Math 122. Figure 1 shows the cost $C(q)$ (in dollars) and revenue $R(q)$ (in dollars) for selling q of the guides.

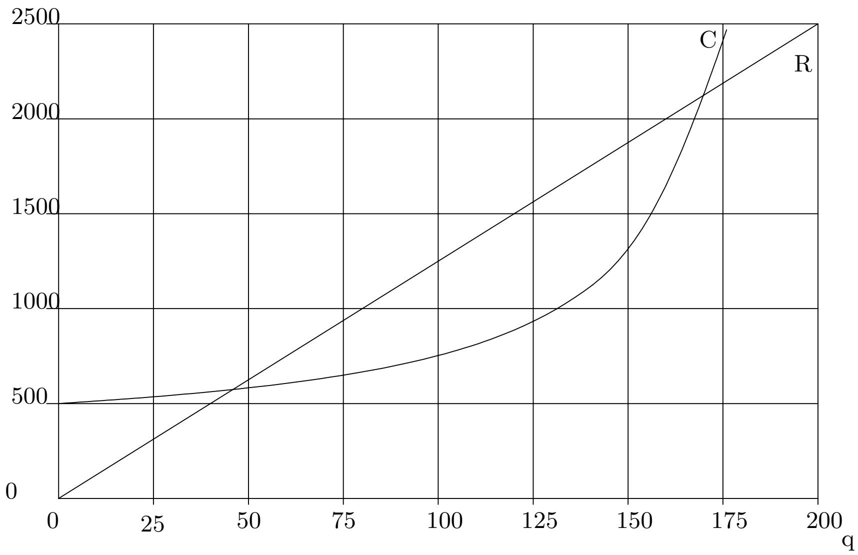

FIGURE 1

(a) What are the startup costs for the students? ________________

(b) At what price are the students selling the quides? ________________

(c) Estimate the cost of producing the $100th$ quide. (That is estimate the marginal cost $C'(100)$.

$C'(100) \approx$ ________________

(d) Estimate the number of quides the students should sell to maximze their profit.

____

(13) (5 points) The variables p and q are related is in the table

p	3	5	7	9
q	14	10	6	2

(a) Explain why the relation between p and q could be linear.

(b) Find p as a function of q. ________________

(c) What if the value of p when $q = 6$? ________________

(14) (5 points) Let $P(t)$ have the values

t	0	1	2
$P(t)$	24	18	13.5

(a) Explain why this could come from an exponential function.

(b) Assuming $P(t)$ is exponential give a formula for $P(t)$.

$P(t) =$ ____________________

(c) What is the half life of $P(t)$? Half life = ____________________

(15) (5 points) If \$1000.00 is invested at 5% interest compounded continuously, how many years does it take to become \$10,000.00?

____

(16) (10 points) If a bee forages for food for t hours a day it expends

$$E = 5t + \frac{a}{t}$$

units of energy where a is a constant. Find the numbers of hours t that minimize the energy.

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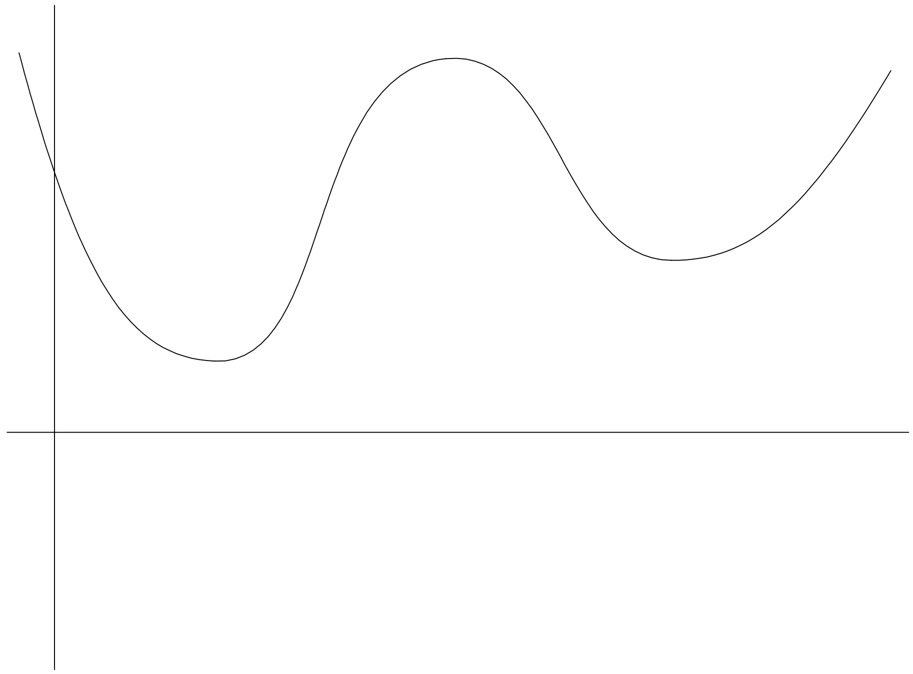
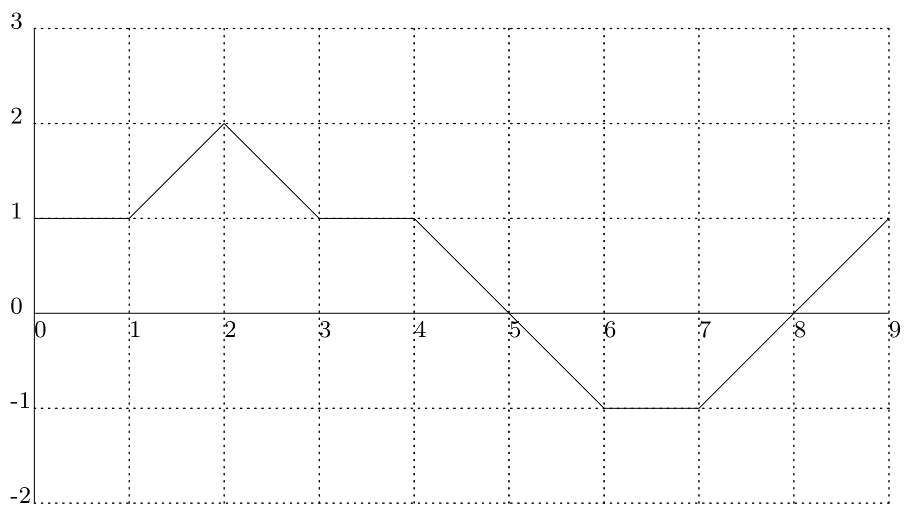
(17) (5 points) If the graph is of $y = f(x)$, draw the graph of $y = f'(x)$ on the same axis.



(18) (10 points) The function $y = f(x)$ has the graph below:



From this compute the following

(a) $\displaystyle\int_0^4 f(x)\,dx$ ____________________

(b) $\displaystyle\int_4^8 f(x)\,dx$ ____________________

(19) (10 points) A car starts moving at time $t = 0$. Its velocity is shown in the following table. Give an upper, lower and best estimate of the distance the car has traveled during the 20 seconds.

t (seconds)	0	5	10	15	20
Velocity (ft/sec)	0	6	18	31	47

Upper estimate ______________________

Lower estimate ______________________

Best estimate ______________________

———————— For Scratch Work ————————

Have a nice winter break.