20 Years of Good Advice

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The following advice is synthesized and distilled from commentaries by judges of the MCM and ICM over the years.

The Model Should Be Based on Research . . .

Teams are increasingly adept at using the Internet to find credible information sources to support their modeling efforts, but there is a good deal of room for improvement in how to incorporate such information into their papers, especially for a team that perceives that it has struck the mother lode of reference sources. Incorporating others' work without diluting one's own effort is challenging. Parroting large portions of technical reports, thereby reducing the team's contribution to simply interpreting someone else's research, is not the solution.

Three uses of existing research are common in technical reports:

- To chronicle the events leading to the approach taken in the current paper and to help the reader understand the context or domain of the problem. This action is typically accomplished in an Introduction or Background section.
- To identify and justify technical parameters needed for the new approach.
- To compare the graphical, symbolic, or numerical results generated by the new modeling approach with those previously identified, so as to examine the benefits or drawbacks of the new approach.

Credible existing research used in these ways does not replace or dilute the current effort but directly supports and strengthens it.

The judges look for evidence whether a team actually did some modeling of its own rather than simply looking up a few equations and trying to shoehorn those into the problem. Experiments can be good to see, too, if appropriate.

Given the time pressure of the contest, a team has to be cautious not to get trapped into adopting a complicated modeling component from existing research without being able to explain clearly its development, its use and limitations, and its impact on the current model. This temptation is the classic red herring of the contest, luring teams into committing to an approach—only to discover late in the process that they are ill-equipped to handle it. Ultimately, evidence of this error appears in the paper as miraculously appearing formulae, unexplained graphics, and tables of data still waiting to be analyzed. Just as in a court of law, the judges consistently find the results of models built on such tenuous foundations difficult to believe.

. . . Must Produce Results . . .

Develop your model—do not just provide a laundry list of possible models. Start with a simple model and then refine it. Also, it is far better to work out one model thoroughly than to present several half-baked approaches. Judges are definitely not interested in a blow-by-blow historical narrative of what you tried that didn't work.

Some papers have noticeable gaps that call into question the validity, veracity, credibility, and applicability of the results presented. Consequently, if a team's principal effort is, say, to construct computer code to simulate an air traffic scenario, *they must present evidence that their code/model actually ran and yielded the information sought.* Analyzing the output of a model provides a basis for determining if the modeling approach chosen was reasonable.

. . . Which Must Be Analyzed . . .

Simply creating an acceptable mathematical representation (system of equations, simulation, differential equations, etc.) of a real-world event is not enough. The representation (model) must be tested to verify that the information that it produces (solutions, simulation output, graphics, etc.) makes sense in the context of the questions asked and the assumptions made. *It is insufficient to present such a representation without this additional evidence.* Once a mathematical model is created, use

- symbolic,
- graphical, and/or
- numerical

methods to exhibit evidence that the model works. Many of the best papers use a combination of these three approaches; some teams write computer code or use spreadsheets, while others use a computer algebra system as their workbench.

. . . and Compared with the Assumptions

Papers reaching the final round of judging paid attention to

- stating their assumptions clearly,
- avoiding making assumptions that are never used or not really needed,
- explaining the impact of each assumption, and
- telling why they felt it was necessary to include it in their model development.

They were also careful not to assume away the challenging and information-relevant portions of the problem posed. It is easy to follow the logical construction of these teams' models and to identify what they were attempting to do. However, sometimes even a very good paper mistakenly places key information in appendices rather than in the section where supporting evidence is desperately needed.

Crucial Elements in an Outstanding Entry

A thorough, informative summary is essential. Your summary is a key component of the paper; it needs to be clear and contain results. It should not say, "Read inside for results." The summary should motivate a judge to read the paper to see how you obtained your results. Even a paper that is otherwise strong can be eliminated in early judging rounds because of a weak summary. Many teams mistakenly form the summary by simply copying parts of the introduction of the paper, which has the different purpose of establishing the background for the problem. On the other hand, the summary should not be overly technical. A long list of techniques can obscure your results; it is better to provide only a quick overview of your approach. Don't merely restate the problem, but indicate how it is being modeled and what was learned from the model. Put into the summary the "bottom-line and managerial recommendation" results—*not a chronological description of what you did.*

Develop a model that people can understand. The model should be easy to follow. While an occasional "snow job" may make it to later rounds of judging, we generally abhor a morass of variables and equations that can't be fathomed. Well-chosen examples enhance the readability of a paper. It is best to work the reader through any algorithm that is presented; too often papers include only computer code or pseudocode for an algorithm without sufficient explanation of why and how it works or what it is supposed to accomplish.

Supporting information is important.

- Figures, tables, and illustrations can help demonstrate ideas, results, and conclusions and thus help sell your model, but *you must refer to these aids in the text of the paper and explain them.* Each such display should have a caption that tells what is in the display, and the display should indicate the measurement units of quantities. Graphs should have scales and axis labels.
- A complete list of references is essential—document where your ideas come from.

Follow the instructions.

- Answer all the required parts and make it clear that you have done so. Attempt to address all major issues in the problem. What the judges pay attention to is whether or not the team engaged the questions asked in the problem. Some teams tell what they know but don't consider the real question—papers missing several elements are eliminated quickly.
- List all assumptions. The problems are deliberately open-ended, and well-posing them is actually part of the problem. Formulating your assumptions is where you pose the problem—making it simple enough to yield to mathematics yet realistic enough to give a credible result.
- State your conclusions and results clearly and make a precise recommendation.

- Don't just copy the original problem statement, but provide us with your interpretation.

Readability Sometimes the quality of writing is so poor that a judge can't follow or make any sense out of the report.

- Make it clear where in the paper the answers are.
- Many judges find a table of contents helpful.
- Your paper needs to be well organized—can a triage judge understand the significance of your paper in 6 to 10 minutes?
- Keep in mind that your audience consists of modeling experts from academia and industry who have only a short time to get the gist of what you did.

More is not necessarily better. If your paper is excessively long (we have had papers over 100 pp long, not including computer program listing), you should probably reconsider the relevance of all factors that you are discussing. Depending on the round of judging, judges have between 5 and 30 min to read a paper. Do not include a single figure, table, or graph that is extraneous to your model or analysis; such additions just distract the judge from discerning what in your paper is important.

Computer Programs

- Clearly define and explain all variables and parameters.
- For a simulation, a single run isn't enough! You must run enough times to have statistically significant output.
- Always include pseudocode and/or a clear verbal description.

Reality Check

- Why do you think your model is good? Against what baseline can you compare/validate it?
- How sensitive is your model to slight changes in the parameters you have chosen? Teams should undertake sensitivity analysis precisely to build credibility in the model,
- Complete the analysis circle: Are your recommendations practical in the problem context?
- Verify as much as you can. Make sanity checks: Is your answer larger than the number of atoms in the known universe? If it is, should it be?
- Use real data if possible.

Triage Judge Pet Peeves

- Tables with columns headed by Greek letters or acronyms that cannot be immediately understood.

- Definitions and notation buried in the middle of paragraphs of text. A bulleted format is easier for the judge.
- Equations without variables defined.
- Elaborate derivations of formulas taken directly from some source. It is better to cite the source and perhaps briefly explain how the formula is derived. It is most important to demonstrate that you know how to use the formulas properly.

Non-technical report of results If a CEO memorandum, press release, or newspaper article is required:

- Be succinct.
- Include "bottom line and managerial results" answers.
- Do not include methods used or equations.

Resources

- All work needs to be original, or else the sources must be cited (including specific page numbers in documented references). A mere list of books or URLs at the end is not sufficient!
- Teams are allowed to use only inanimate resources—no real people or people consulted over the Internet.
- Surf the Web but cite sites for information that you use.
- Use high-quality references. Peer-reviewed journals, books, and government Websites are preferable to individuals' Websites or blogs.

How to Proceed

- **Read the problem statement carefully.** Words implying actions (design, analyze, compare, etc.) are keys to sections that your paper should contain. Organize the paper into sections corresponding to the parts of the problem; if certain broad topics are required, begin with an outline based on them.
- **Make your paper easy to read.** Number the pages, tables, figures, and equations; check the spelling; and use a large-enough font size.
- **Define terms** that a reader might find ambiguous, particularly any term used in the model that also has a common prose meaning.
- Address sensitivity to assumptions as well as the strengths and weaknesses of the model. These topics should be covered separately in sections of their own. **Go back to your list of assumptions and make sure that each one is addressed.** This is your own built-in checklist aiding completeness; use it.

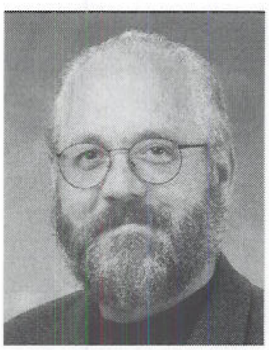
- **Your summary should state the results that you obtained, not just what you did.** Keeping the reader in suspense ("we will develop another model later . . . ") is a good technique in a novel, but it simply frustrates the judges.
- **Do more than is asked.**
- **Write informally, write well.** In many student-written papers, as a colleague puts it, "nobody ever does anything—things just happened." Too common is a chronological narrative in the stilted no-person passive past tense ("Then it was discovered that the data could be fitted by a fifth-degree polynomial . . . "). Much better is a story of first-person present-tense activity ("We fit a fifth-degree polynomial to the data . . . ").

References

Cline, Kelly. 2005. Kelly Cline's Guide to the MCM. `http://web.carroll.edu/kcline/mcm.html` .

Specific instructions on "how to attack the MCM," including a timetable, by a participant who improved steadily (Successful Participant ⟶ Honorable Mention ⟶ Outstanding) over three years and now coaches teams at Carroll College, Montana.

About the Authors



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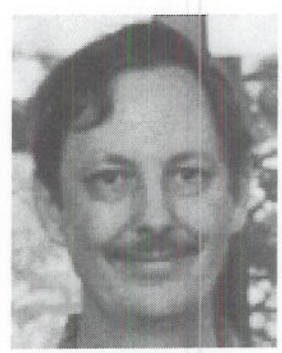
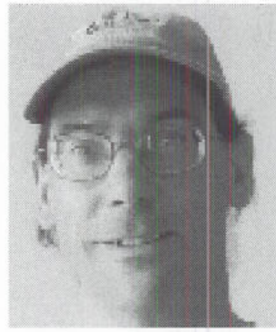


Jerry Griggs is a graduate of Pomona College and MIT (Ph.D., 1977). Since 1981, he has been at the University of South Carolina, where he is Professor of Mathematics. His research area is combinatorics and graph theory. He has published more than 75 papers and supervised 12 doctoral and 15 master's students. He is the editor-in-chief of the *SIAM Journal on Discrete Mathematics* and serves on the board that oversees the Canada/USA Mathcamp. He has been an MCM judge since 1988 and is the author of four MCM problems.



Mark Parker is Associate Professor of Mathematics at Carroll College. He earned his Ph.D. in Applied Mathematics (combinatorial optimization) from the University of Colorado–Denver. In addition to teaching at Carroll, Eastern Oregon University, and the U.S. Air Force Academy, he spent eight years as a systems simulation and analysis engineer at Lockheed-Martin. After four years as a final-round MCM judge, he enjoys the role of mentor and coach for the Carroll MCM and ICM teams.

Paul Boisen is an Applied Research Mathematician at the National Security Agency. He works a wide range of problems in a dynamic environment and encourages MCM participants to contact him if they join NSA. He was a Fulbright Professor to the Hungarian Academy of Sciences, a Postdoctoral Fellow at MSRI, a graduate student at the University of Chicago (Ph.D. mathematics, 1990), and a student in the first Budapest Semester in Mathematics (Spring 1985). He was a final-round MCM judge 1998–2001 and a head triage judge 2001–2002.

William P. Fox is Professor and Chair of the Dept. of Mathematics at Francis Marion University. He received his M.S. (operations research) from the Naval Postgraduate School and his Ph.D. (operations research and industrial engineering) from Clemson University. He was formerly at the U.S. Military Academy. He makes conference presentations and has co-authored several texts on mathematical modeling. He directs the High School Mathematical Contest in Modeling and has authored numerous MCM/ICM problems.

Mike Tortorella is a Research Professor of Industrial and Systems Engineering at Rutgers University and Managing Director of Assured Networks, LLC. He retired after 26 years from Bell Laboratories as a Distinguished Member of the Technical Staff. He has a Ph.D. (Purdue University) in mathematics. His research interests include stochastic flow networks, information quality and service reliability, and numerical methods in applied probability. Mike has been an MCM judge since 1993 and has particularly enjoyed MCM problems with a practical flavor of mathematics and society. He enjoys amateur radio, the piano, and cycling; he is a founding member of the Zaftig Guys in Spandex road cycling team.

Since the start of the MCM, Paul Campbell has enjoyed editing for *The UMAP Journal* the Outstanding papers and the commentaries on them.