

TOPICS IN MATHEMATICAL LOGIC APPLICABLE TO NUMBER THEORY

MATH 768 N

Spring 2005

MW 4:40 pm to 5:55 pm

LeConte 316

Instructor: George F. McNulty

Resources: Hilbert's Tenth Problem

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MIT Press

Model Theory

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Prerequisite: Graduate Standing

At the turn of the 20th century, David Hilbert posed 23 problems scattered across the broad range of mathematics that, in large measure, touched the heart of mathematics as it has developed in the intervening years.

Hilbert's Tenth Problem concerned Diophantine equations. These are polynomial equations in several variables with integer coefficients.

Hilbert's Tenth Problem. *Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined by a finite number of operations whether the equation is solvable in rational integers.*

Of course, Hilbert was not the only one to offer conjectures that have challenged and motivated mathematicians. Emil Artin posed the following conjecture.

Artin's Conjecture. *For each positive integer d there exists a finite set Y of primes such that for every prime $p \notin Y$, every polynomial $f(t_0, \dots, t_{n-1})$ of degree d over the field \mathbb{Q}_p of p -adic numbers with zero constant term and $n > d^2$ has a nontrivial zero in \mathbb{Q}_p .*

Both of these problems were solved using methods from mathematical logic. This course will develop the tools from mathematical logic that have proven useful in tackling problems in number theory. The goal of the course is to come to terms with the solution to Hilbert's Tenth Problem. The tools from logic come from the theory of computable functions and lead to further results. For example, the solution to Hilbert's Tenth Problem leads to a polynomial in 10 variables with integer coefficients whose positive values are exactly the prime numbers. Such a description of the prime numbers was commonly assumed to be impossible (or at least inaccessible).

It is anticipated that MATH 788 (Topics in Number Theory by Way of Logic) will be offered in the following semester. This course will focus on tools from the theory of models and the resolution of Artin's Conjecture.