Problem. *Suppose that the various line segments joining pairs of 6 noncollinear points in the plane are each colored either red or blue. Show that there must be a **monochromatic** triangle, a triangle with all its edges the same color.*

Solution. Label the points A, B, C, D, E and F. There are 5 line segments from point A, namely $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$ and $\overline{AF}$. Each of these is colored either red or blue. By the pigeonhole principle, at least 3 of these edges are colored the same color, say red (an analogous argument works if the color is blue). Then by relabeling the points if necessary, we may suppose that the edges $\overline{AB}$, $\overline{AC}$ and $\overline{AD}$ are all red. If $\overline{BC}$ is also colored red, then $\triangle ABC$ is a monochromatic red triangle. Similarly, if $\overline{BD}$ or $\overline{CD}$ is colored red, then $\triangle ABD$ or $\triangle ACD$, respectively, is a monochromatic red triangle. Thus, if each of $\triangle ABC$, $\triangle ABD$ and $\triangle ACD$ is not monochromatic, then we must have that $\overline{BC}$, $\overline{BD}$ and $\overline{CD}$ are all colored blue. In this case, we obtain that $\triangle BCD$ is a monochromatic blue triangle. We deduce that, in any case, there must be a monochromatic triangle. $\blacksquare$