MATH 574, NOTES 7
PRACTICE PROBLEMS FOR TEST 2

(1) How many 4-permutations of the set $\{1, 2, 3, 4, 5, 6\}$ contain the number 2?

(2) The number 12 written in base 2 is $(1100)_2$ which ends in 2 zeroes. In how many zeroes does the number 50! end when it is expressed in base 2?

(3) How many solutions are there to the equation

$$x_1 + x_2 + x_3 = 10$$

if each x_j is to be an integer from $\{0, 1, 2, \ldots, 10\}$?

(4) Let $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_r$ be a complete list of distinct subsets of $\{1, 2, \ldots, n\}$.

(a) Explain why $r = 2^n$.

(b) If $|\mathcal{A}_j|$ denotes the number of elements in the subset $\mathcal{A}_j$, then what is the value of

$$|\mathcal{A}_1| + |\mathcal{A}_2| + \cdots + |\mathcal{A}_r|?$$

(5) Calculate $\displaystyle\sum_{k=1}^{n} \frac{\binom{n}{k}}{2^k}$ in closed form. (Note that the sum begins with $k = 1$ and not $k = 0$.)

(6) Prove that $\displaystyle\sum_{k=0}^{n} \frac{\binom{n}{k}}{k+1} = \frac{2^{n+1}-1}{n+1}$.

(7) (a) Recall that

$$1 + x + x^2 + \cdots = \frac{1}{1-x} \quad \text{if } |x| < 1.$$

Give a closed form expression for the sum $1 + 2x + 3x^2 + 4x^3 + \cdots$ that holds for $|x| < 1$.

(b) Calculate $\displaystyle\sum_{k=0}^{\infty} \frac{k}{2^k}$.