MATH 574, NOTES 4
THE PIGEONHOLE PRINCIPLE

Examples:

(1) A person has 20 socks in a drawer, 10 of which are black and 10 of which are brown. How many socks must he pull out to assure he has two socks the same color? How many socks must he pull out to assure he has two socks the *opposite* color?

(2) Given five "lattice" points in the plane, show that the line segment containing some two of them has midpoint a lattice point. (Note that the same is not true if only four lattice points are given.)

(3) Show that in a group of two or more people who shake hands (no one shakes hands with his-or-herself and no one shakes hands with the same person twice), some two of them shook the same number of hands.

(4) Suppose that the various line segments joining pairs of 6 noncollinear points in the plane (how many are there?) are each colored either red or blue. Show that there must be a triangle formed with all of its edges the same color.

(5) Given any $n+1$ integers from the set $\{1, 2, \ldots, 2n\}$, show that there are at least two of them, say a and b, such that a divides b.

(6) Show that for every real number α, there are integers a and b with $b \neq 0$ such that

$$\left|\alpha - \frac{a}{b}\right| < \frac{1}{b^2}.$$

(Discuss good rational approximations in general.)

(7) Given a sequence of n^2+1 distinct real numbers, there exists a subsequence of length $n+1$ which is either increasing or decreasing. (Example: The sequence $7, 5, 6, 1, 10, 2, 9, 3, 8$ of 8 numbers does not have such a subsequence of length 4 but if 4 or 0 or 11 is added to the end, it does.)