MATH 574, NOTES 10
MISCELLANEOUS CONSIDERATIONS

Examples:

(1) A plane is cut into two regions by a line. A plane can be cut into a maximum of 4 regions by two lines. What is the maximal number of regions that a plane can be cut into by n lines?

(2) What is the maximal number of regions that a block of cheese can be cut into by n planar slices of a knife? What if we keep the pieces in their original position (rather than stacking them)?

(3) My wife and I attended a party with four other married couples. Some handshaking took place. Of course, no one shook hands with his or herself and no one shook hands with his or her spouse. Also, no one shook hands with the same person more than once. Afterwards, I asked each person how many hands he or she shook. To my surprise each of the nine others shook a different number of hands. How many hands did my wife shake at the party?

(4) Two people play a game. We begin with $N = 0$. Each person takes turns choosing a number from $\{1, 2, 3\}$, adding it to N to form a new number N, and announcing what the new N is. The winner is the first person to get the number N to be ≥ 15. How should one play this game? How does the strategy of the game change if 15 is replaced by 24?

(5) Describe the game of Nim. What's the best strategy for playing the game?

(6) If each edge of a complete graph on 66 vertices is colored one of 4 different colors, why must the edges joining some 3 vertices form a monochromatic triangle?

(7) If each edge of a complete graph on n vertices is colored one of 5 different colors, then the edges joining some 3 vertices must form a monochromatic triangle. What is the minimal value for n you can think of which guarantees this statement is true?