MATH 574, NOTES 1
PROOFS BY CONTRADICTION

Examples:

(1) $\sqrt{2}$ is irrational.

(2) An irrational number to an irrational power can be rational.

(3) e is irrational.

(4) There are infinitely many primes.

(5) The arithmetic-geometric mean inequality holds $\left(\dfrac{a+b}{2} \geq \sqrt{ab} \text{ for } a > 0 \text{ and } b > 0\right)$.
(Is a proof by contradiction really appropriate here?)

(6) The real numbers are *uncountable*.

(7) If a finite set of points on the plane have the property that the line through any two of the points also passes through a third point, then the points all lie on a straight line.

Homework:

(1) (a) Prove that $\sqrt{3}$ is irrational.

(b) Since $\sqrt{4}$ is rational (it's 2), one cannot prove that $\sqrt{4}$ is irrational. But one can try to. Try doing a proof that $\sqrt{4}$ is irrational using an argument similar to what you did in part (a) and what was done in class for $\sqrt{2}$. We'll discuss what's wrong with the proof.

(2) Prove that $\log_2 3$ is irrational.

(3) Prove that $\sqrt[3]{3}$ is irrational.

(4) Prove that the sum of a rational number and an irrational number is irrational.