Math 532: Quiz 3

Name ______________________________

Axiom 1. There exist exactly 3 points.
Axiom 2. Given any 2 distinct points, there exists exactly one line passing through the 2 points.
Axiom 3. Given any line, there is a point not on the line.
Axiom 4. Any two lines intersect in at least one point.

1. Finish the proof below that, for an axiomatic system with the axioms above, each line has exactly two points on it.

Proof: Let ℓ be a line. By Axiom 3, there is a point ______________________.

By Axiom 1, there are exactly 3 points. Therefore, ℓ has ______ ($\leq$ or $\geq$) 2 points on it. Assume

ℓ has ______ ($<$ or $>$) 2 points on it. Then there are at least ______ points *not* on ℓ. Let ℓ' be a

line passing through 2 of these points, say A and B. We know ℓ' exists by Axiom ______.

Observe that $\ell' \neq \ell$ since ______________________________________.

By Axiom 4, ℓ and ℓ' have at least one point, say P, in common. We know that $P \neq A$ and

$P \neq B$ since ________________________________. Since Axiom 1 indicates

that there are exactly 3 points, these 3 points must be ____________________. This

contradicts Axiom ______ since __________________________________.

Therefore, ℓ must have exactly 2 points on it, finishing the proof. ■