Math 532: Homework 6

(1) For $A = (1, 2)$, $B = (7, -1)$ and $C = (2, 4)$. Calculate each of the following:

(i) $B - A$

(ii) $(B - A)^2$

(iii) $C - 2A$

(iv) $(B - A)(C - A)$

(v) Is $\triangle ABC$ a right triangle?

(2) Let A, B, and C be 3 noncollinear points. Let D be the intersection of the altitude in ΔABC drawn from A with the altitude drawn from B. Then $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{BC}$, and $\overrightarrow{BD}$ is perpendicular to $\overrightarrow{AC}$. Recall that the dot product of perpendicular vectors is 0. Use this to show that all 3 altitudes of ΔABC are concurrent.

(3) For a triangle ΔABC, let M_A be the midpoint of $\overline{BC}$, M_B be the midpoint of $\overline{AC}$, and M_C be the midpoint of $\overline{AB}$. Along $\overline{BC}$ draw a perpendicular at M_A, along $\overline{AC}$ draw a perpendicular at M_B, and along $\overline{AB}$ draw a perpendicular at M_C.

(a) Show that these 3 perpendiculars share a common point.

(b) If D is the point in (a), show that D is equidistant from A, B, and C.

(Note that there is an easy way to do problem (3) without making use of vectors, but try the problems using vectors anyway.)

(4) Suppose ΔABC and $\Delta A'B'C'$ are such that $\overleftrightarrow{AB}$ and $\overleftrightarrow{A'B'}$ are parallel, $\overleftrightarrow{BC}$ and $\overleftrightarrow{B'C'}$ are parallel, and $\overleftrightarrow{AC}$ and $\overleftrightarrow{A'C'}$ are parallel. Show that either the lines $\overleftrightarrow{AA'}$, $\overleftrightarrow{BB'}$, and $\overleftrightarrow{CC'}$ are all parallel or they all intersect at a common point.