Math 532: Homework 4

The following problems are about affine planes of order n. The axioms for an affine plane of order n are:

Axiom A1. There exist at least 4 distinct points no 3 of which are collinear.

Axiom A2. There exists at least 1 line with exactly n points on it.

Axiom A3. Given any 2 distinct points, there exists exactly one line passing through the 2 points.

Axiom A4. Given any line ℓ and any point P not on ℓ, there is exactly 1 line through P that does not intersect ℓ.

(1) Show that an affine plane of order n does not satisfy the principle of duality.

(2) Show that in an affine plane of order n, each point has exactly $n + 1$ lines passing through it.

(3) Show that in an affine plane of order n, each line has exactly n points on it.

(4) Show that in an affine plane of order n, each line is parallel to exactly $n - 1$ lines.

(5) Show that in an affine plane of order n, there are exactly n^2 points and exactly $n^2 + n$ lines.