

Math 532: Homework 1

The first 7 problems deal with an axiomatic system consisting of the following axioms:

Axiom 1. There exist exactly 3 points.

Axiom 2. Given any 2 distinct points, there exists exactly one line passing through the 2 points.

Axiom 3. Given any line, there is a point not on the line.

Axiom 4. Any two lines intersect in at least one point.

- (1) Prove that the axiomatic system is consistent.
- (2) Is the axiomatic system independent? Justify your answer.
- (3) Prove that any two distinct lines intersect in exactly one point.
- (4) Prove that each line has exactly two points on it.
- (5) Prove that there are exactly three lines.
- (6) Is the axiomatic system complete? Justify your answer.
- (7) Does the principle of duality hold for the axiomatic system? Justify your answer.

The next several problems deal with an axiomatic system consisting of the axioms below. These problems came from a test by the instructor from a previous Math 532 course.

Axiom 1. There exist at least 3 noncollinear points.

Axiom 2. Given any 2 distinct points, there exists exactly one line passing through the 2 points.

Axiom 3. Given any 2 distinct lines which are not parallel, there does not exist a third line parallel to both of the given lines.

Axiom 4. Given any line ℓ and any point P not on ℓ , there is at least one line parallel to ℓ that P lies on.

- (8) Is the axiomatic system consistent? Justify your answer.
- (9) Is the axiomatic system independent? Justify your answer.
- (10) Is the axiomatic system complete? Justify your answer.
- (11) Does the principle of duality hold for the axiomatic system? Justify your answer.
- (12) Prove that there exist 4 points no 3 of which are collinear.