**Analysis Seminar, Wednesday, April 15, 12:00, LC 312**
**A Local Fatou Theorem for Harmonic Functions on the Hyperbolic Ball**
**Manfred Stoll**

**Abstract** Let $\mathbb{B}$ denote the unit ball in in $\mathbb{R}^n$, $n \geq 2$, with hyperbolic metric $ds^2 = 4(1-|x|^2)^{-2}|dx|^2$. The Laplacian $\Delta_h$ with regard to the hyperbolic metric is given by

$$\Delta_h f(x) = (1-|x|^2)^2 \Delta f(x) + 2(n-2)(1-|x|^2)\langle x, \nabla f(x)\rangle,$$

where $\Delta f$ denotes the ordinary Laplacian of $f$ and $\nabla f$ is the gradient of $f$. For $\zeta \in \partial\mathbb{B}$, $\alpha > 1$, $\Gamma_\alpha(\zeta)$ denotes the nontangential approach region

$$\Gamma_\alpha(\zeta) = \{x \in \mathbb{B} : |x-\zeta| < \alpha(1-|x|)\}.$$

Also, for $E \subset \partial\mathbb{B}$ and $\alpha > 1$, let $\Omega(E,\alpha) = \bigcup_{\zeta\in E} \Gamma_\alpha(\zeta)$. In the seminar we will prove the following local Fatou theorem.
**Theorem** *If $E \subset \partial\mathbb{B}$ is measurable, $\alpha > 1$, and $U$ is a bounded function on $\Omega(E,\alpha)$ such that $\Delta_h U = 0$. Then*

$$\lim_{\substack{x\to\zeta \\ x\in\Gamma_\beta(\zeta)}} U(x) \qquad \textit{exists}$$

*for a.e. $\zeta \in \partial E$ and $\beta > 1$.*

This result was originally proved by I. Privalov in 1923 for harmonic functions in the unit disc. It was subsequently extended to harmonic functions in the upper half–space in $\mathbb{R}^n$ by A. P. Calderon. The proof will make use of several classical results in analysis, including the Hahn–Banach Theorem, Riesz Representation Theorem, and the Ascoli–Arzela Theorem.