Speaker: Kevin Beanland (Washington & Lee)

Title: ℓ_p structure in general Banach spaces.

Abstract: Given a Banach space X with a Schauder basis (e_i) it is natural to ask if one can find an infinite sequence of block vectors $x_1, x_2, x_3, ...$ that are equivalent the the unit vector basis of ℓ_p for some $1 \leq p < \infty$ or c_0. It turns out that this is not always possible. The corresponding finite dimensional version of this problem, however, does have a positive answer. That is, for each $\epsilon > 0$ and Banach space X with a Schauder basis (e_i) there is a p with $1 \leq p \leq \infty$ so that for each $n \in \mathbb{N}$ there is a finite sequence of block vectors $x_1, x_2, ..., x_n$ that is $(1+\epsilon)$ equivalent to ℓ_p^n. This is a deep result of Jean-Louis Krivine that appeared in the Annals in the early 1970s. These p's that appear in a given space are now called Krivine p's of that space. A Krivine p of a Banach space X is stabilized on a subspace Y of X if it is a Krivine p of Y and for every subspace Z of Y, p is a Krivine p of Z. Any stabilized Krivine p set for a Banach spaces is closed in $[1, \infty]$. A long standing conjecture held that the this set had to be either a singleton or a closed interval. In recent work with Dan Freeman and Pavlos Motakis we construct several Banach spaces that strongly disprove this conjecture. In particular for any finite set F of $[1, \infty]$ there is a Banach space so that each stabilized Krivine set is F. In this talk we will discuss the history of this problem and the construction of the counterexample.