Speaker: Reid Harris (USC)

Title: Hausdorff Measure and Convex Functions

Abstract: On a metric space, we can define an outer measure $\langle^\alpha$ by

$$\langle^\alpha(E) = \lim_{\delta \to 0} \left(inf \left\{ \sum_i (\text{diam } A_i)^\alpha : E \subset \bigcup_i A_i, \text{diam } A_i < \delta \right\} \right).$$

This measure is a generalization of Lebesgue measure $\mathcal{L}$, in the sense that it coincides with $\mathcal{L}$ on Borel subsets of $\mathbb{R}^n$. Given a convex subset C of $\mathbb{R}^n$, we can find a decomposition of C into a union $C_0 \cup \ldots C_n$ where each C_i, $i < n$ has zero Hausdorff measure in analogy with the decomposition of polytopes into it's faces. In order to do this, the notion of the subdifferential of a convex function needs to be outlined.