

Problem Set 1

MATH 776, Fall 2007, Cooper

Expiration: Thursday September 13

Problems are ranked 0-5 based on their difficulty. The number of points awarded for a **fully correct, rigorous** answer turned in before the expiration date above is $2^{\text{difficulty}}$. These problems are in addition to the problems in Diestel §1, which correspond to difficulties of 1,2, and 3, depending on whether they are marked with a (-), no mark, or a (+), respectively.

1. (3) Prove that $(\sqrt{2})^k \leq R(k, k) \leq 4^k$ for $k \geq 3$.
2. (2) Show that a 2-colored complete graph on 6 vertices (i.e., K_6) actually contains 2 monochromatic triangles (i.e., K_3 's).
3. (4) Compute $R(3, 3, 3)$.
4. (4) Prove the Infinite Ramsey Theorem: If K_ω (a complete graph whose vertices are the natural numbers) is 2-colored, then there exists a monochromatic sub- K_ω .
5. (3) Alice and Bob play a game in which they alternate coloring edges of a K_6 red or blue. The first player to complete a monochromatic triangle *loses*. Can the first player always win?